1. The Language and Logic of Proofs

A. Language and Symbols

Use the corresponding Mathematical Language/symbols to shorten long sentences:

$\in -$ *Belongs to or element of*

$\cup -$ *Union*

$\cap -$ *Intersection*

$\subset -$ *Subset*

$\subseteq -$ *Subset or Equal to*

$\mathbb{N} -$ *set of Natural Numbers*

$\mathbb{Z}$ *or* $\mathbb{J} -$ *set of Integers*

$\mathbb{Q} -$ *set of Rational Numbers*

$\mathbb{R} -$ *set of Real Numbers*

$\mathbb{C} -$ *set of Complex Numbers*

$\Rightarrow -$ *Implies*

$\Leftrightarrow$ *or iff* $-$ *Equivalent to or if and only if*

$\equiv -$ *Congruent*

$\Vert\Vert -$ *Similar*

$\therefore -$ *Therefore*

$\because -$ *Since or Because*

$: -$ *So that*

$\forall -$ *for All*

$\exists -$ *there Exists*

$\neg P$ *or* $\sim P$ *or* P'*or* $\bar{P} -$ *not* P

B. Characteristics of Statements

1. Each statement can be considered either True of False.
2. The evidence used to establish a statement is true is the proof.
3. A statement can be proven true by ensuring EVERY possible example in the case is true. It only takes ONE Counterexample to disprove the statement (only takes ONE Counterexample to show the statement to be false).

Example

1. Determine whether the following statements are true of false, given reasons where necessary:

a) 6 is an even number.

b) 8 is an odd number.

c) The square of a number is even.

d) 4 is an even number and 3 is an odd number.

e) 4 is an even number or 3 is an odd number.

f) 4 is an even number and 6 is an odd number.

g) 4 is an even number or 6 is an odd number.

C. Types of Statements

1. Quantifiers
   - Words that indicate a quantity include
     Universal Quantifier – "for all" (denoted by ∀)
     Existential Quantifier – "for some/at least one/there exists" (denoted by ∃)

     eg. The statement "For all real values of x, there is at least one real value of y such that the sum of x and y is 6" can be written as "$\forall x \in \mathbb{R}, \exists y \in \mathbb{R}: x + y = 6$"

Example

1. Rewrite the following statements using mathematical symbols and determine whether the statements are true or false, justify your answer.

a) The square of any integer is greater than the integer itself.

b) There is a real number which, when multiplied by 5, gives an answer of 0.

c) The sum of the squares of any two real numbers is less than the product of the numbers.